FEA for the NHMFL 15T system

Author: David Max *MA DPhil*

This example presents results of finite element analysis (FEA) calculations for a superconducting magnet design described in the literature.

The FEA problem was solved using the **Calculix FEA** package.

The magnet was built at the National High Magnetic Field Laboratory (NHMFL) in Florida, USA. Coil dimension data for the magnet system are given by Markiewicz et al 1994 [1], who also present detailed results for stress and strain in the coils and external binder.

Purpose

The aim of this study was to assess whether fitting a stainless steel binder to the outside of one of the magnet sections was effective in reducing stress in that section.

Contents

Figures

Tables

Introduction: construction of the NHMFL 15T system

The NHMFL 15 tesla magnet consists of two coaxial solenoids, each of winding length 500 mm.

- The **inner** section of the magnet is wound from niobium-tin conductor.
- The **outer** is niobium-titanium.

The inner section has a (stainless steel) **external binder** on the outside of the coil to take up some of the electromagnetic stress.

The question in this study is to ask how effective the external binder was.

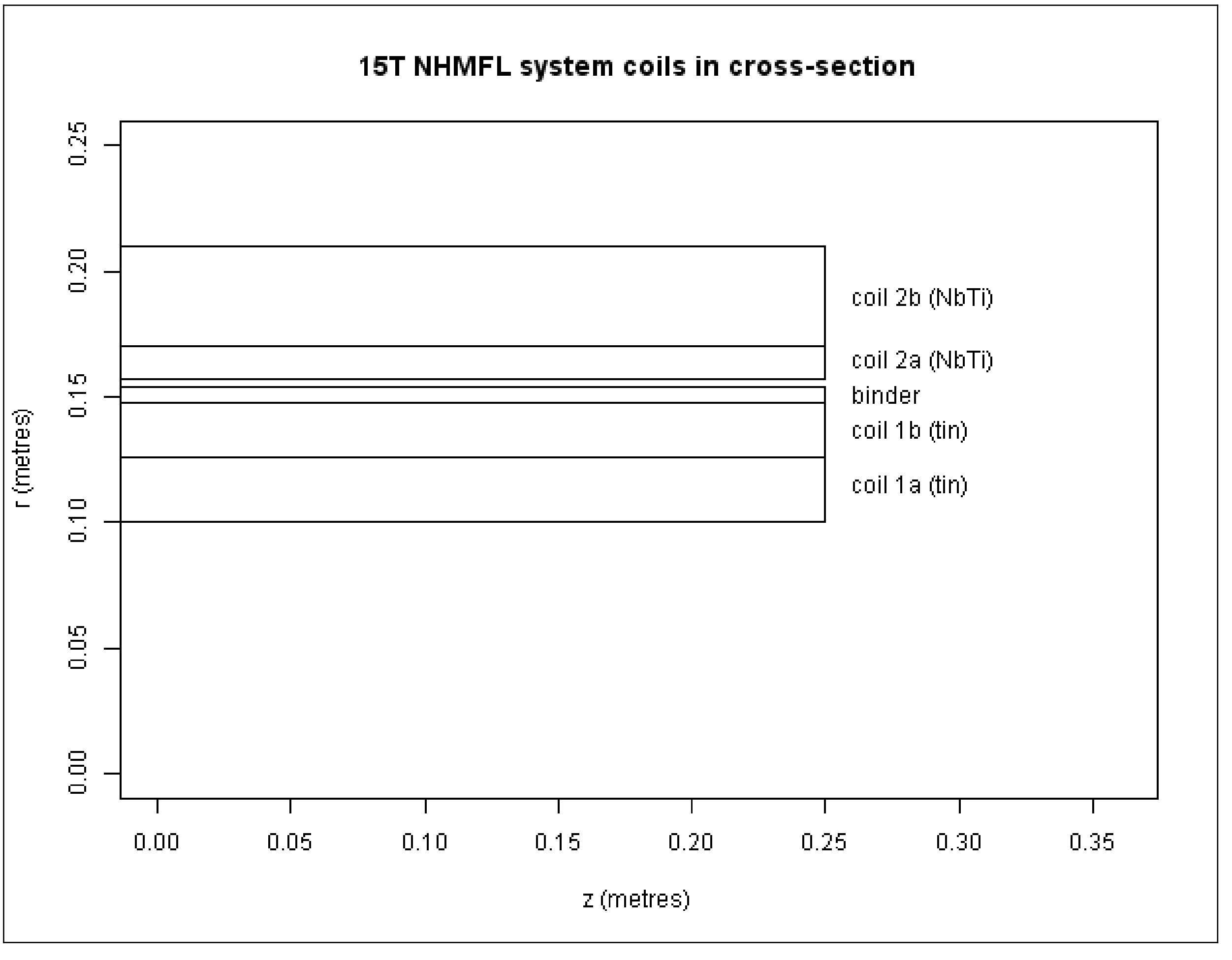


Figure 1 Coils in cross-section, NHMFL 15T system

Main results in Markiewicz et al. 1994

Peak hoop stress in the Nb_3Sn wire is shown to be around 200 MPa (Figure 2 in the paper, where it is labelled 'tangential stress'). Hoop stress in the external binder is much higher at around 330 MPa.

Figure 3 in the paper shows maximum hoop strain in the wire was about 3.1 millistrains (3.1e-03). This is around the level at which niobium-tin short-sample wire performance is maximised [2], owing to the relief of the compressive pre-strain that results from the high-temperature reaction process. The NHMFL design may thus be optimised in this respect. Conditions may have been adjusted by the designers to take maximum advantage of the performance of the expensive niobium-tin wire.

Assumptions for the Calculix FEA finite element analysis model

For this case study, only electromagnetic forces are considered, and thermal contraction effects have been ignored. No information about winding tension is given in the paper so this also must be neglected. For comparison, models were solved using two packages, **Calculix FEA** and **LISA FEA**.

Only a brief outline summary of the Calculix results for the tin section (coils 1a and 1b) will be given here.

Binder (TEST362) and no-binder (TEST363) model comparison

Two models were constructed, one with the reinforcing binder (**TEST362**) and one without (**TEST363**), to investigate the effectiveness of fitting the binder.

In these models, the coils, and in TEST362, the binder, were represented as a regular array of eight-node quadrilateral elements (Calculix type '**CAX8**').

A table of Lorentz force per unit volume (axial and radial components) was computed for the elements. The forces on each element were split between nodes by 'mass lumping' [3] and the contributions (from up to four elements) summed for each node. Next the results of the mass lumping process were built into a Calculix 'input deck' (a .inp file), along with node coordinates, node numbers for each element, materials properties etc. The orthotropic material properties given in the Markiwewicz et al paper can be specified using the Calculix 'card' *ELASTIC,TYPE=ENGINEERING CONSTANTS.

The work of constructing the Calculix FEA input deck was automated using **Fortran-77** and an **R** script.

The Calculix FEA model was then solved using the Calculix **ccx** command. (A point of interest here is that, behind the scenes, Calculix converts the eight-node quadrilateral elements into 20-node brick elements with a circumferential thickness of 2 degrees.) After solving the model, the results then require post-processing.

Note on orientation of axes

For calculations on axisymmetric magnet systems, the *z*-axis is typically chosen to be the axis of rotational symmetry for the magnet coils. Often a cylindrical coordinate system is used, with ρ (the radius) being the distance away from this *z*-axis.

On the other hand, the standard elements in Calculix FEA assume the axis of rotational symmetry is the *x*-axis, and *z* then is equivalent to distance in the hoop direction.

Table 1 explains the relationship between conventional and Calculix FEA axes.

Table 1 Axis conventions

item	usual convention	equivalent here using Calculix FEA
axis of rotational symmetry	z	*x*
radial offset from axis of rotational symmetry	ρ	*y*
hoop direction	Φ (azimuthal angle)	*z*
hoop stress	$\sigma_{\Phi\Phi}$	szz
axial stress	σ_{zz}	sxx
radial stress	$\sigma_{\rho\rho}$	syy

R graphics for model results

The Calculix graphical post-postprocessor **cgx** produces nice graphics but these are of fixed dimensions (width and height) and the quality deteriorates if the size is changed.

Some post-processing of the Calculix results was therefore also done in R to produce colour-coded plots using R's **grid** package [4]. Part of this process involved extracting some of the results tables from the Calculix .frd and .dat output files. Some of this work relies on R functions in the **stringr** package.

Main results of models TEST362, TEST363

The results of the model TEST362, which includes the binder, are in reasonable agreement with those in the paper, though they are not identical. Results are shown in Table 2.

The deviations may reflect differences in model assumptions. Certain assumptions are necessary in order to simplify model construction. For example, here the binder is assumed to be fixed in the axial and radial directions *relative* to the outer surface of the underlying niobium-tin coil. A better model might be to allow the two surfaces to slide relative to each other, but this would entail introducing contact into the model and would be a significant complication.

Table 2 Computed stresses and strains, with and without binder

field	units	TEST362				TEST363	
		in binder		in coils		in coils	
		min.	max.	min.	max.	min.	max.
sxx (radial stress)	MPa	-10.0	2.9	-14.8	4.8	0.1	11.8
syy (axial stress)	MPa	-200.1	7.2	-37.2	2.7	-68.2	-0.2
szz (hoop stress)	MPa	141.0	287.9	47.1	**215.8**	86.5	**254.9**
sxy (shear stress)	MPa	-2.5	8.5	-1.6	9.0	-2.3	0.3
svm (von Mises stress)	MPa	138.5	426.1	51.6	**218.3**	86.7	**274.1**
EXX (radial strain)	(none)	-0.0005	-0.0001	-0.0018	-0.0005	-0.0019	-0.0007
EYY (axial strain)	(none)	-0.0015	-0.0002	-0.0017	-0.0003	-0.0026	-0.0006
EZZ (hoop strain)	(none)	0.0007	0.0018	0.0008	**0.0033**	0.0013	**0.0041**

Hoop stress (szz)

For peak **hoop stress** and **hoop strain**, finite element analysis model TEST362 and the Markiewicz et al paper are in fair agreement. Hoop stress results are as follows: model TEST362, 215.8 MPa; Markiewicz et al 1994, ~205 MPa.

Hoop strain

Hoop strain in the niobium-tin wire is an important design consideration as the short-sample performance of the wire is strain-sensitive.

Agreement between the Calculix FEA model and the Markiewicz paper for hoop strain is good: model TEST362, 0.0033; paper: ~0.0031.

The overall pattern of radial stress and strain on the magnet centreplane is also similar.

Axial stress (sxx)

Agreement is not so good for computed **axial stress**. Model TEST362 shows a peak of -200 MPa in the binder, whereas the paper gives a peak of -150 MPa.

Von Mises stress (svm)

The high computed axial stress is a significant contributor to the high level of von Mises stress in the binder (426 MPa).

Peak von Mises stress in the adjacent outer layers of the coil is much lower at around 140 MPa. (See Figure 9, Figure 10 in the Appendix).

For superconducting magnet designs, peak magnet von Mises stress may be a good indicator of overall design viability.

Figure 2 TEST362: hoop stress, szz

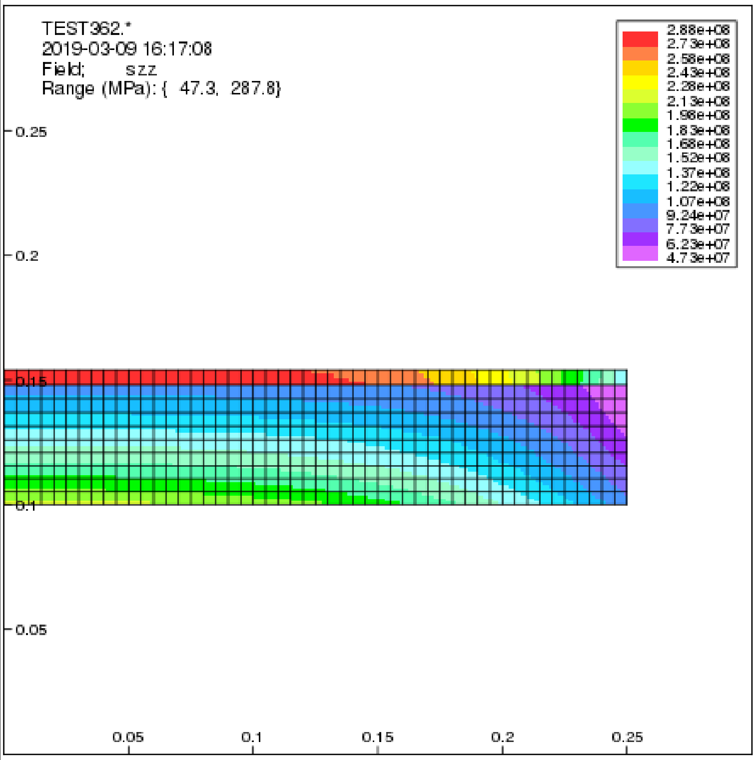
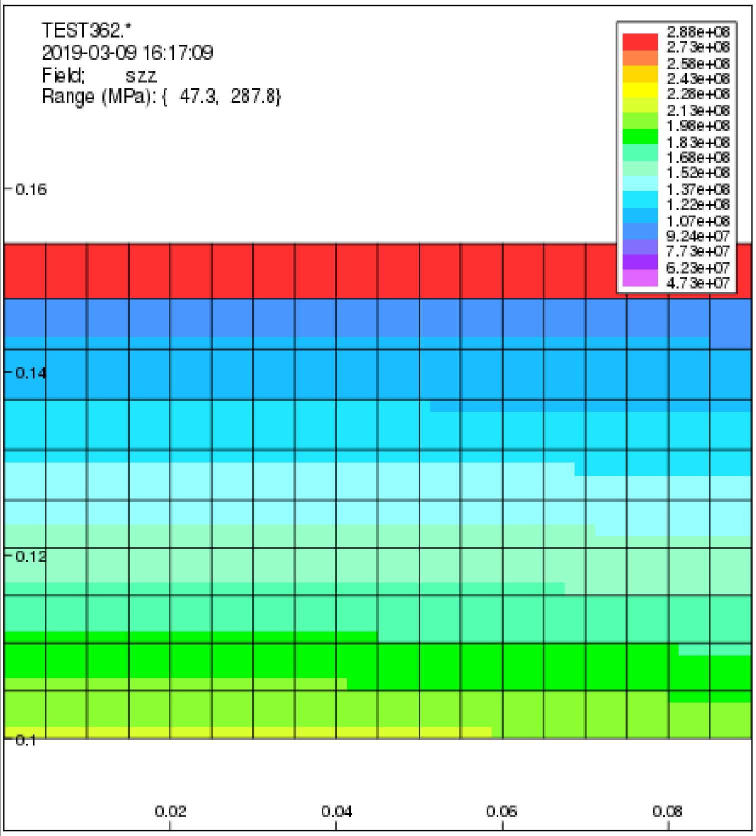


Figure 3 TEST362: hoop stress near centreplane



Shear stress

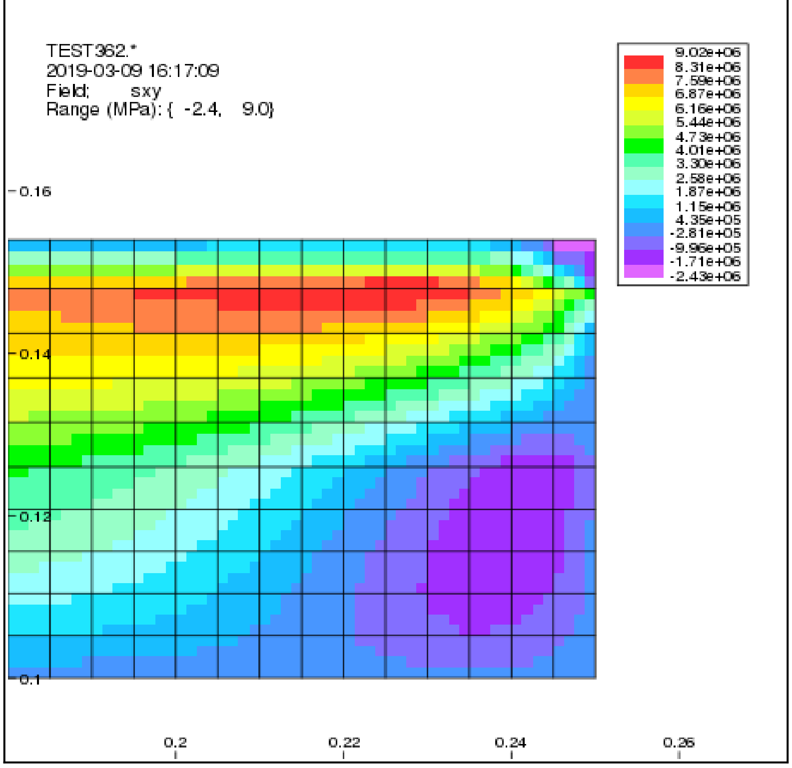
A useful ‘sanity check’ on the model is the levels of computed shear stress in the hoop directions (sxz and syz).

By symmetry these quantities (in Calculix-speak, ‘entities’) should be zero, or very small, reflecting numerical precision limits. This indeed is the case (see Figure 13, Figure 14 in the Appendix).

In one region near the end of the coil, shear stress **sxy** near the coil-binder interface is quite high (9.0 MPa). Just possibly, this may be high enough to cause local slippage between the coil and external binder, if the two were not bonded together in any way.

If slippage did occur, local frictional heating might be sufficient to initiate quenches in this region, although this is a low-field, and thus relatively stable, part of the coil. Shear stress near the centreplane, where there is a very sharp discontinuity in axial stress between coil and binder, is negligible.

Figure 4 TEST362: shear stress sxy near coil end



Conclusion: did the external binder work well?

Comparing models TEST362 and TEST363, several points indicate that the external binder was very successful and served its purpose well.

1) The binder has reduced peak hoop strain in the coil from 4.1 millistrains to 3.3 millistrains. Thus the tin section operates under approximately 'optimal strain' conditions [2].

2) Peak von Mises stress in the coil falls from 274 MPa (without binder) to 218 MPa (with binder). Even the resulting level with the binder would be quite high for, say, an MRI magnet.

3) Hoop stress falls from 254.9 MPa (without binder) to 215.8 MPa (with binder).

4) Peak axial stress in the coil has almost halved (without binder: -68.2 MPa; with binder: -37.2 MPa).

5) The highest levels of stress have effectively ‘migrated’ into the binder.

References

[1] Markiewicz, W.D., Vaghar, M.R., Dixon, I.R. & Garmestani, H. 1994. IEEE Transactions on Magnetics, **30(4)**, 2233-2236.

[2] Cheggour, N. & Hampshire, D.P. 1999. IEEE TRANSACTIONS ON APPLIED SUPERCONDUCTIVITY **9(2)**, 2517-2520.

[3] Zienkiewicz, O.C., Taylor, R.L. & Zhu, J.Z. 2013. *The Finite Element Method: Its Basis and Fundamentals.* Elsevier, Amsterdam.

[4] Murrell, P. 2011. *R Graphics Second Edition.* CRC Press, Boca Raton.

Appendix

Plots of various computed quantities from the models are presented over the next few pages.

Axial stress

Regions of the coil are under axial tensional stress in much of the interior of the coil (the colours correspond to positive numbers of computed sxx).

A region near the end plane is under compressive stress -- levels are reasonably modest.

Figure 5 TEST362: axial stress, sxx

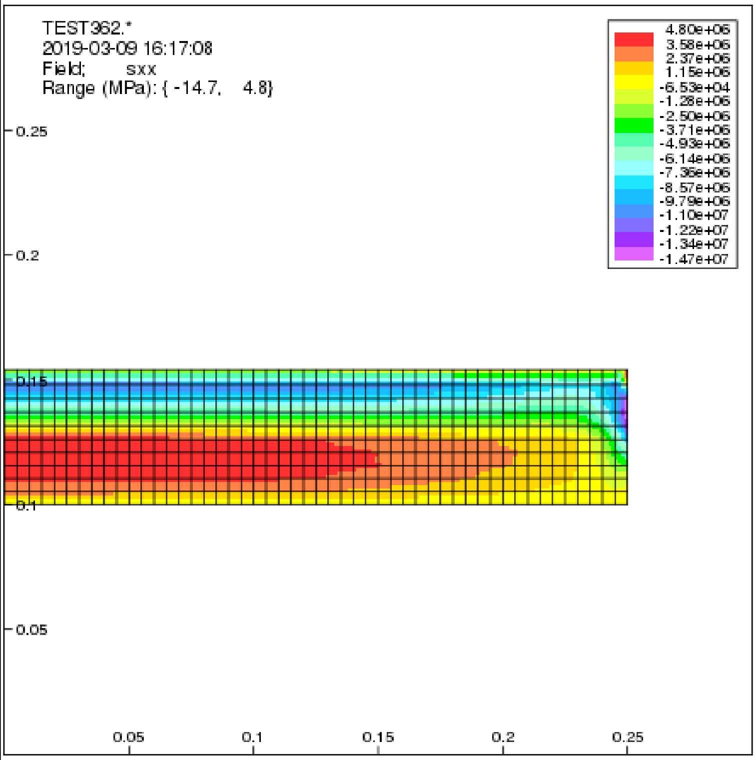
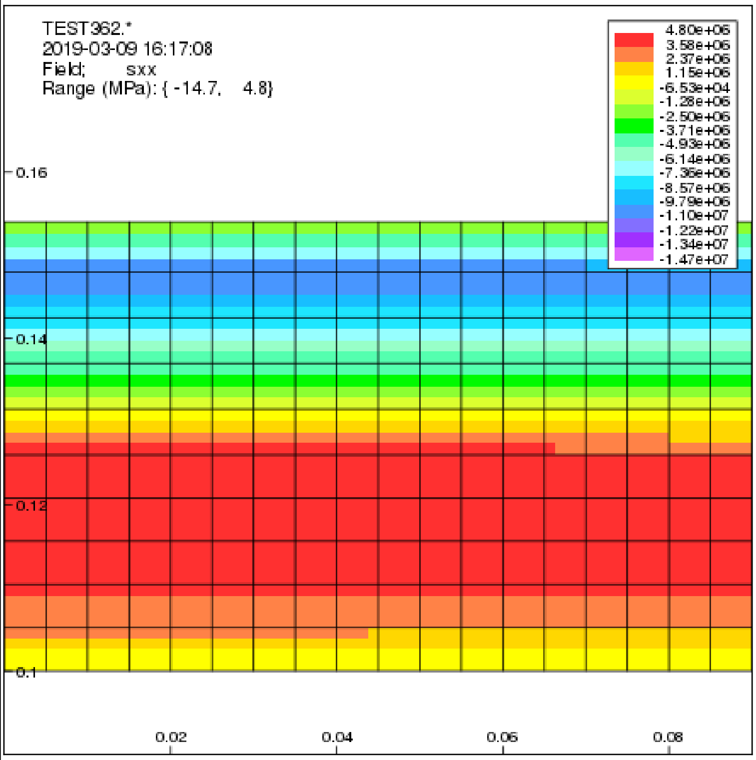


Figure 6 TEST362: axial stress near centreplane



Radial stress

Radial stress in the coils is quite modest but high in the binder, which is under strong radial compression near the centreplane.

Von Mises stress (svm)

Near the centreplane, von Mises stress (a kind of total stress) is high in the binder partly owing to high levels of axial stress here.

Figure 9 TEST362: von Mises stress, svm

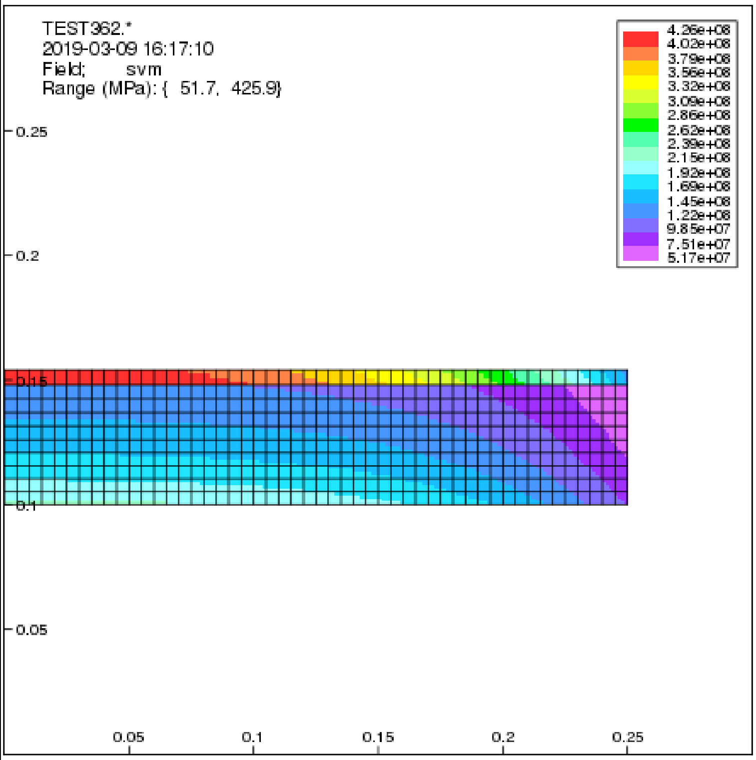
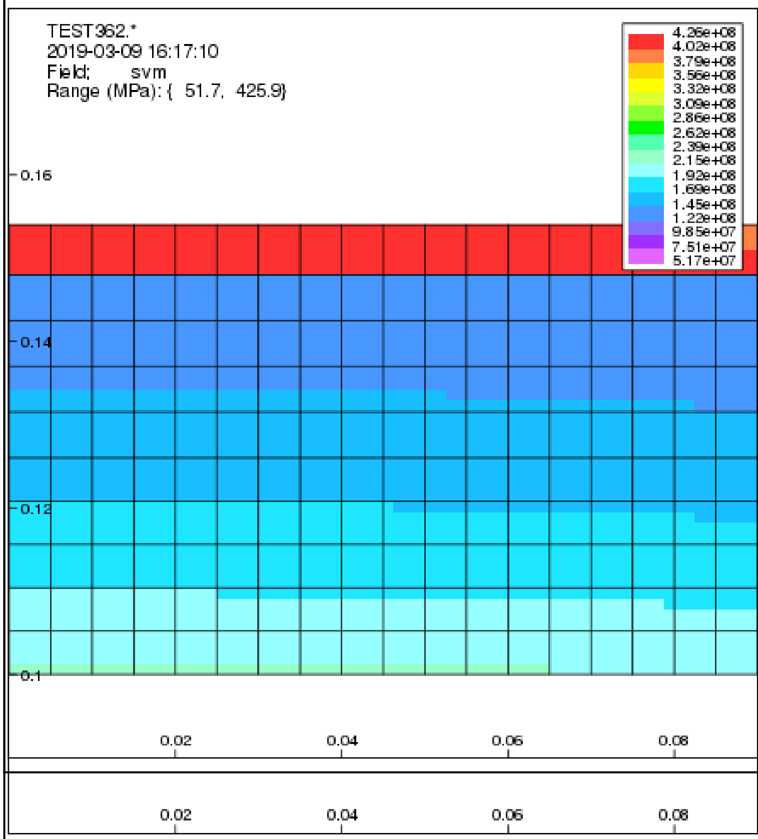


Figure 10 TEST362: von Mises stress near centreplane



Shear stress sxy

Shear stress near the coil end is quite high.

This will have the effect of tending to tear the layers apart axially.

Whether the levels are high enough to cause problems is unknown.

Figure 11 TEST362: shear stress, sxy

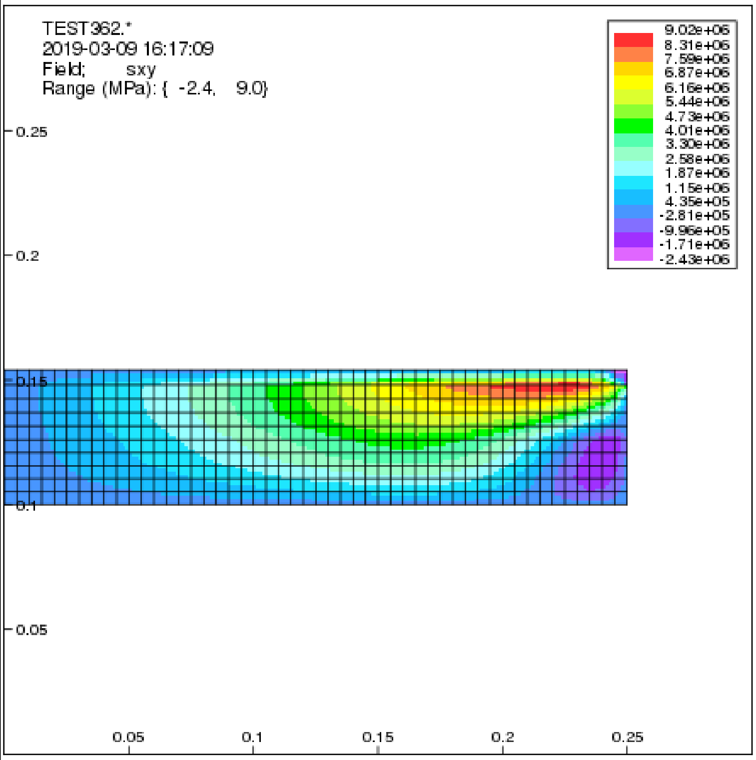
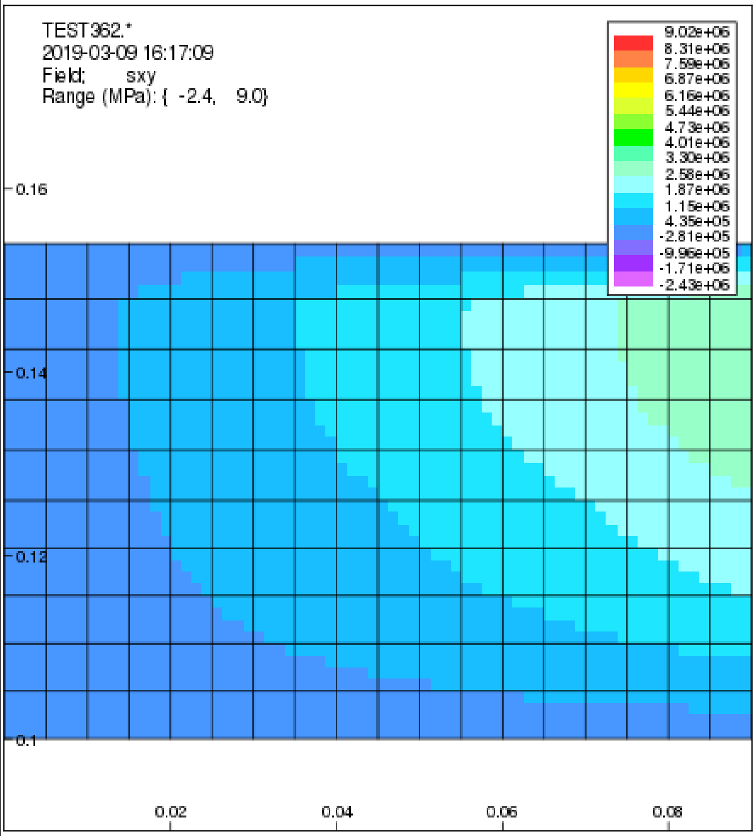


Figure 12 TEST362: shear stress sxy near centreplane



Shear stress sxz

Owing to the cylindrical symmetry of the coils, shear stress terms involving the hoop direction (either sxz or syz) should be zero, as seen here. (Note the exponents are negative, e.g. top colour band is 7.22e-07).

Figure 13 TEST362: shear stress, sxz

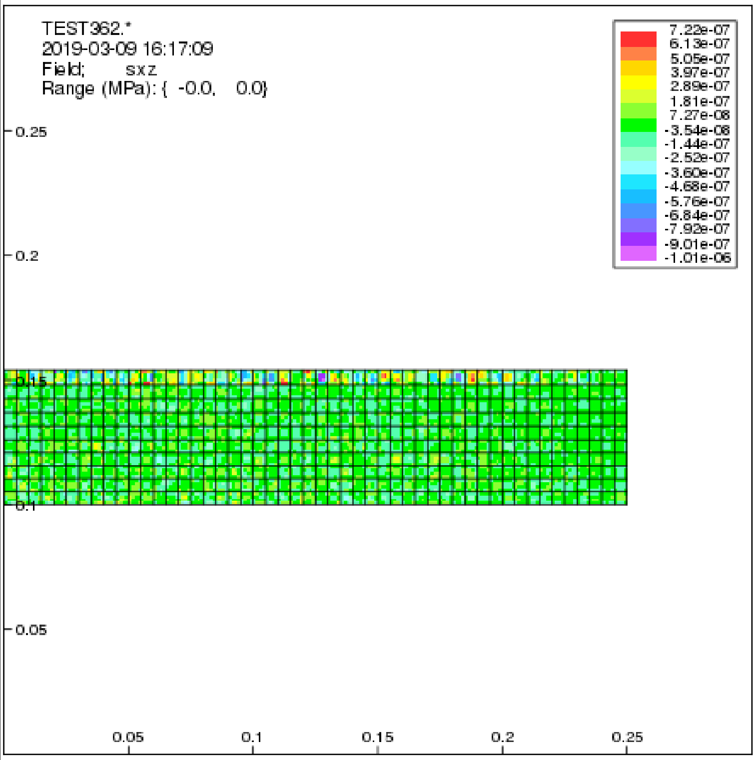


Figure 14 TEST362: shear stress sxz near centreplane